Final Worksheet

1. Create a truth table for $\big(P \to (\neg Q \vee R)\big) \wedge (R \vee Q)$

2. Let x, y be real number, show that if $x \neq y$, then $2x + 4 \neq 2y + 4$

3. Let n be an integer, prove that if $3n$ is odd, n is odd

4. Consider the statement “the equation $x^2 + 2x = 15$ has a unique solution”, describe a universe in which this statement is true

5. $\forall x, \big(x \in A \Leftrightarrow \exists\, n, (n \in \mathbb{Z} \land x = 2n)\big)$

   Express the above statement in a sentence and describe what is this set A

6. Prove if the product, xy, is not a rational number, then x or y must be an irrational number

7. Provide counterexample to the following statements:
   a. Every odd number is prime
   b. Every prime number is odd
   c. For every real number x, we have $x^2 > 0$
   d. For every real number $x \neq 0$, we have $\frac{1}{x} > 0$

8. Let A, B, C be the following sets:

   $A = \{(x, y) \in \mathbb{R}^2 : x - y = 0\}$

   $B = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$

   $C = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 = 0\}$.

   Prove that $A \cup B = C$

9. Prove that $\{n^2 + n + 1 : n \in \mathbb{N}\} \subseteq \{2n + 1 : n \in \mathbb{N}\}$

10. Prove or disprove: If $A \cup B = A \cup C$, then $B = C$

11. Prove $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

12. Write the set $\mathcal{P}\big(\{\emptyset, \{\emptyset\}\}\big)$ explicitly

13. Define three relations on $\mathbb{R}$ by $x \sim y$ if and only if there exists $n \in \mathbb{Z}$ such that

a. $x, y \in [n, n+1]$

b. $x, y \in [n, n+1)$

c. $x, y \in [n, n+2)$

Prove or disprove each of the above relations is an equivalence relation.

14. Define relation on $\mathbb{R}$ by $x \sim y$ if and only if there exists $n \in \mathbb{Z}$ such that $x, y \in [n, n+1)$.

Prove that the indexed collection of equivalence classes $\{C_x : x \in \mathbb{R}\}$ is a partition of $\mathbb{R}$

15. Define $f: \mathbb{Z} \to \mathbb{N}$ by $f(x) = |x|$, prove or disprove that f is a function

16. Define $f: \mathbb{R}^2 \to \mathbb{R}$ by $f\big((x, y)\big) = x$

a. Prove or disprove that f is injective (1-1)?

b. Prove or disprove that f is surjective (onto)?

17. Let A be a nonempty set with $|A| = n$ and let $\alpha \in A$. Prove that $A\backslash\{\alpha\}$ is finite and

$|A\backslash\{\alpha\}| = n - 1$

18. Prove $|\mathbb{N}| < |\mathbb{R}|$

19. Prove $|\mathbb{Z}| \leq |\mathcal{P}(\mathbb{Z})|$

(Note: This statement is not equivalent to $|\mathbb{Z}| < |\mathcal{P}(\mathbb{Z})|$)